# Strong & Weak Goldbach Conjectures Proved Side-by-Side

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# Abstract

After proving the strong Goldbach conjecture (viXra:2006.0226), the author, in this paper, covers both the weak Goldbach conjecture and the strong Goldbach conjecture. The strong Goldbach conjecture states that every even integer greater than 4 can be expressed as the sum of two odd primes. The weak Goldbach conjecture states that every odd integer greater than 7 can be expressed as the sum of three odd primes. The approach in the coverage of the weak Goldbach conjecture is similar to the approach used in proving the strong conjecture. However, two approaches for producing Goldbach partitions for the weak conjecture are covered. In the first approach, one applies the principles used in finding partitions for the strong conjecture. Beginning with the partition equation, $9 = 3 + 3 + 3$, and applying the addition of a 2 to both sides of this equation, and subsequent equations, one obtained Goldbach partitions for 26 consecutive odd integers. In the second approach, one produces the partitions from the partitions of the strong conjecture by adding a 3 to both sides of a strong conjecture partition equation. For the strong conjecture, one will begin with the partition equation $6 = 3 + 3$, and apply the addition of 2 to both sides of the equation to produce the partition for the next even number, 8. From the partition equation, $8 = 5 + 3$, one will repeat the 2-addition process to obtain the partition for the next even integer, 10. From the partition for 10, the process can continue indefinitely. This repetitive process was compared to the repetitive process in compound interest calculations. It is shown that given an equation for a Goldbach partition, one can produce a Goldbach partition for any even integer greater than 4 as well as produce a partition for any odd integer greater than 7. A consequent generalized procedure also produced Goldbach partitions for non-consecutive even and non-consecutive odd integers. In addition to directly producing partitions of the strong conjecture, one can also produce partitions of the strong conjecture from the partitions of the weak conjecture and vice versa. Formulas derived for the Goldbach partitions show that every even integer greater than 4 can be written as the sum of two odd prime integers; and also that every odd integer greater than 7 can be written as the sum of three odd prime integers. Importantly, in addition to showing that the Goldbach conjectures are true, this paper shows how to produce Goldbach partitions.

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# Preliminaries

## Introduction

The following is from the first page of the author's high school practical physics note book: Science is the systematic observation of what happens in nature and the building up of body of laws and theories to describe the natural world. Scientific knowledge is being extended and applied to everyday life. The basis of this growing knowledge is experimental work. To prove Goldbach conjecture, one would be guided by the approach for producing Goldbach partitions.

## Finding Goldbach Partitions for Even Integers

For communication purposes, let $E_n$ be a positive even integer, and let $P_r$ and $P_s$ be two odd prime integers, where the subscript $n$ is an even integer and the subscripts $r$ and $s$ are odd prime integers, and $E_n = P_r + P_s$.

The main principle for finding a Goldbach partition for a consecutive even integer from a known partition is the application of the addition axiom which states that if equal quantities are added to equal quantities, the sums are equal. One will begin with the Goldbach partition, $6 = 3 + 3$, and apply the addition axiom to this equation to obtain partitions for larger even integers. In obtaining partitions for consecutive even integers, one will add 2 to both sides of the equation. For the next consecutive even integer, one will add a 2 to the present even integer, and one will also add a 2 to the right side of the equation. For the addition of a 2 to the right side of the equation, there are three main possibilities.

**Possibility 1:** (desirable)**:** The addition of a 2 to any of the terms results in two prime integers.

**Possibility 2:** One will inspect the two terms, $P_r$ $P_s$ to determine which term would still be prime when a 2 is added to it, and add to it accordingly.

**Possibility 3:** If on adding a 2, to any of the terms, say, $P_r$ one obtains a composite integer, one may add another 2 or more 2's until one obtains a prime integer; but for compensation, one would have to subtract the extra 2 or the extra 2's (added) from the other term $P_s$, noting that one is adding a single 2 to the left side and a single 2 to the right side of the equation. For examples, if one repeats the addition of 2 once, one will subtract a single 2 from the other term. If one repeats the addition twice, one will subtract 4 from the other term. If one repeats the 2-addition a third time, one will subtract a 6 from the other term. With experience, one would be able to determine which term to add a 2 or 2;s to in order to avoid the extra 2-addition. For the difference in the subtraction, the resulting difference cannot be less than 3. The difference must be a prime number. An important skill for finding Goldbach partitions is changing composite integer sums to prime integer sums.

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# Condensed Goldbach Partitions for Even Integers Consecutive Descendants

The following table condenses the processes involved in partition production and will show that every even integer greater than 4 can be expressed as the sum of two odd primes, because the process will not terminate and can continue indefinitely. Review the instructions on page 3. One will begin with the partition equation $6 = 3 + 3$, and apply the addition of 2 to both sides of the equation to produce the partition for the next even number, 8. From the partition equation, $8 = 5 + 3$, one will repeat the 2-addition process to obtain the partition for the next even integer, 10. From the partition for 10, the process will continue indefinitely and every even integer would be partitioned as the sum of two odd primes.

## Table 1

**Basis:** $n = 2k = 6$,

| | |
|---|---|
| **1**. $6 = 3 + 3$ | **26.** $56 = 53 + 3 = 37 + 19$ |
| **2.** $8 = 5 + 3$ | **27.** $58 = 53 + 5 = 41 + 17$ |
| **3**. $10 = 7 + 3$ | **28.** $60 = 53 + 7 = 43 + 17 = 41 + 19$ |
| **4**. $12 = 7 + 5$ | **29.** $62 = 59 + 3 = 43 + 19$ |
| **5**. $14 = 7 + 7 = 11 + 3$ | **30.** $64 = 59 + 5 = 47 + 17$ |
| **6.** $16 = 11 + 5 = 13 + 3$ | **31.** $66 = 59 + 7 = 47 + 19$ |
| **7.** $18 = 13 + 5$ | **32.** $68 = 61 + 7$ |
| 8. $20 = 13 + 7$ | **33.** $70 = 67 + 3 = 59 + 11$ |
| **9.** $22 = 17 + 5 = 11 + 11$ | **34.** $72 = 67 + 5 = 59 + 13$ |
| **10.** $24 = 19 + 5 = 13 + 11$ | **35.** $74 = 71 + 3 = 61 + 13 = 67 + 7$ |
| **11.** $26 = 19 + 7 = 23 + 3 = 13 + 13$ | **36.** $76 = 71 + 5 = 59 + 17$ |
| **12.** $28 = 23 + 5 = 17 + 11$ | **37.** $78 = 71 + 7 = 61 + 17$ |
| **13.** $30 = 23 + 7 = 17 + 13$ | **38.** $80 = 73 + 7 = 61 + 19 =$ |
| **14.** $32 = 29 + 3 = 21 + 11$ | **39.** $82 = 79 + 3 = 59 + 23$ |
| **15.** $34 = 31 + 3 = 29 + 5$ | **40.** $84 = 79 + 5 = 61 + 23$ |
| **16.** $36 = 31 + 5 = 29 + 7$ | **41.** $86 = 79 + 7$ |
| **17.** $38 = 31 + 7$ | **42.** $88 = 83 + 5$ |
| **18.** $40 = 37 + 3 = 29 + 11$ | **43.** $90 = 83 + 7$ |
| **19.** $42 = 37 + 5 = 29 + 13$ | **44.** $92 = 89 + 3$ |
| **20.** $44 = 37 + 7 = 31 + 13$ | **45.** $94 = 89 + 5$ |
| **21.** $46 = 41 + 5 = 29 + 17$ | **46.** $96 = 89 + 7$ |
| **22.** $48 = 43 + 5 = 29 + 19$ | **47.** $98 = 79 + 19$ |
| **23.** $50 = 43 + 7 = 31 + 19$ | **48.** $100 = 83 + 17$ |
| **24.** $52 = 47 + 5 = 29 + 23$ | **49.** $102 = 83 + 19$ |
| **25.** $54 = 47 + 7 = 31 + 23$ | **50.** $104 = 97 + 7$ |

Above, beginning with the partition equation , $6 = 3 + 3$, the partition equations #2 to #50 were produced. Also, knowing partition #5, above, one can also obtain partition #39 and vice versa. In the table, there are fifty Goldbach partitions.

# Condensed Goldbach Partitions for Odd Integers Consecutive Descendants

Two tables are presented for the production of the partitions for the odd integers. The first table will be constructed similarly as the condensed table on the previous page., and will show that every odd integer greater than 7 can be expressed as the sum of three odd primes, because the process will not terminate, and can continue indefinitely. The second table (in Table 3) will be constructed from the table of partitions for even integers from the author's paper proving the strong conjecture (viXra:2006.0226). In the table below, one will begin with the partition equation $9 = 3 + 3 + 3$, and apply the addition of 2 to both sides of the equation to produce the partition for the next odd number, 11. From the partition equation, $11 = 5 + 3 + 3$, one will repeat the 2-addition process to obtain the partition for the next odd integer, 13. From the partition for 13, the process could continue indefinitely and every odd integer would be partitioned as the sum of three odd primes. Note that an odd number can have two or more different partitions.

## Table 2

**Basis:** $n = 2k + 1 = 9$,

| | |
|---|---|
| **1**. $9 = 3 + 3 + 3$ | ⇓ Begin |
| **2**. $11 = 5 + 3 + 3$ | ⇐ (Add 2 to 9, and add 2 to the first 3. |
| **3.** $13 = 7 + 3 + 3$ | ⇐ (Add 2 to 11, and add 2 to 5. |
| **4.** $15 = 7 + 5 + 3$ | ⇐ (Add 2 to 13, and add 2 to the first 3. |
| **5.** $17 = 7 + 7 + 3$ | ⇐ (Add 2 to 15, and add 2 to 5. keep the 3 |
| **6.** $19 = 7 + 7 + 5$ | ⇐ (Add 2 to 17, and add 2 to 3 |
| **7**. $21 = 7 + 7 + 7$ | ⇐ (Add 2 to 19, and add 2 to 5. |
| **8.** $23 = 11 + 5 + 7$ | ⇐ (Add 2 to 21,add 4 to the first 7, subtract 2 from the second 7. |
| **9.** $25 = 13 + 5 + 7$ | ⇐ (Add 2 to 23, add 2 to 11. |
| **10.** $27 = 13 + 7 + 7$ | ⇐ (Add 2 to 25,add 2 to 5 |
| **11.** $29 = 17 + 5 + 7$ | ⇐ (Add 2 to 27, add 4 to 13, subtract 2 from the first 7. |
| **12**. $31 = 19 + 5 + 7$ | ⇐ (Add 2 to 29, add 2 to 17,keep 5 and 7 |
| **13.** $33 = 19 + 7 + 7$ | ⇐ (Add 2 to 31, add 2 to 5. |
| **14**. $35 = 23 + 5 + 7$ | ⇐ (Add 2 to 33, add 4 to 19,subtract 2 from the first 7. |
| **15**. $37 = 23 + 7 + 7$ | ⇐ (Add 2 to 35, and add 2 to 5 |
| **16**. $39 = 23 + 11 + 5$ | ⇐ (Add 2 to 37, add 4 to the first 7 and subtract 2 from the second 7. |
| **17**. $41 = 23 + 13 + 5$ | ⇐ (Add 2 to 39, add 2 to 11 |
| **18**. $43 = 23 + 13 + 7$ | ⇐ (Add 2 to 41, add 2 to 5. |
| **19**. $45 = 29 + 13 + 3$ | ⇐ (Add 2 to 43, add 6 to 23, subtract 4 from7 |
| **20**. $47 = 29 + 13 + 5$ | ⇐ (Add 2 to 45, and add 2 to 3. |
| **21**. $49 = 29 + 13 + 7$ | ⇐ (Add 2 to 47, and add 2 to 5. |
| **22**. $51 = 31 + 13 + 7$ | ⇐ (Add 2 to 49, add 2 to 29. |
| **23**. $53 = 31 + 17 + 5$ | ⇐ (Add 2 to 51, add 4 to 13 and subtract 2 from 7. |
| **24**. $55 = 31 + 17 + 7$ | ⇐ (Add 2 to 53, add 2 to the 5 |
| **25**. $57 = 31 + 19 + 7$ | ⇐ (Add 2 to 55, add 2 to 17, keep the 7. |
| **26**. $59 = 31 + 17 + 11$ | ⇐ Add 2 to 57, add 4 to 7, subtract 2 from 19 |

**About the above explanations**: When one adds a 4, one has added an extra 2, and one subtracts appropriately a 2 from one of the other addends. If one adds a 6, one has added an extra 4 which must be subtracted from one (or both) of the other addends, noting that the addition and subtraction of a 2 or 2's is to produce only primes on the right side of the partition equation.

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## Weak Conjecture Partitions Constructed from Strong Conjecture Partitions

(Consecutive Descendants)

The following table contains partition equations for both the strong conjecture and the weak conjecture. The weak conjecture on the right was obtained from the strong conjecture on the left by merely adding a 3 to both sides of each partition equation on the left. Both sides of the table show that the partition production process will not terminate and can continue indefinitely. Thus, for the strong conjecture, every even integer greater than 4 can be expressed as the sum of two odd primes; and for the weak conjecture, every odd integer greater than 7 can be expressed as the sum of three odd primes. Review the instructions on page 3. For the strong conjecture, one will begin with the partition equation $6 = 3 + 3$, and apply the addition of 2 to both sides of the equation to produce the partition for the next even number, 8. From the partition equation, $8 = 5 + 3$, one will repeat the 2-addition process to obtain the partition for the next even integer, 10. From the partition for 10, the process can continue indefinitely. For the weak conjecture, one will add a 3 to both sides of each corresponding partition equation of the strong conjecture column.

**Table 3**

| Strong Goldbach Conjecture | Weak Goldbach Conjecture |
|---|---|
| **1**. 6 = 3 + 3 | **1**. 9 = 3 + 3 + 3 |
| **2.** 8 = 5 + 3 | **2.** 11 = 5 + 3 + 3 |
| **3**. 10 = 7 + 3 | **3**. 13 = 7 + 3 + 3 |
| **4**. 12 = 7 + 5 | **4**. 15 = 7 + 5 + 3 |
| **5**. 14 = 7 + 7 = 11 + 3 | **5**. 17 = 7 + 7 + 3 = 11 + 3 + 3 |
| **6.** 16 = 11 + 5 = 13 + 3 | **6.** 19 = 11 + 5 + 3 = 13 + 3 |
| **7.** 18 = 13 + 5 | **7.** 21 = 13 + 5 + 3 |
| 8. 20 = 13 + 7 | 8. 23 = 13 + 7 + 3 |
| **9.** 22 = 17 + 5 = 11 + 11 | **9.** 25 = 17 + 5 + 3 = 11 + 11 + 3 |
| **10.** 24 = 19 + 5 = 13 + 11 | **10.** 27 = 19 + 5 + 3 = 13 + 11 + 3 |
| **11.** 26 = 19 + 7 = 23 + 3 | **11.** 29 = 19 + 7 + 3 = 23 + 3 + 3 |
| **12.** 28 = 23 + 5 = 17 + 11 | **12.** 31 = 23 + 5 + 3 = 17 + 11 + 3 |
| **13.** 30 = 23 + 7 = 17 + 13 | **13.** 33 = 23 + 7 + 3 = 17 + 13 + 3 |
| **14.** 32 = 29 + 3 = 21 + 11 | **14.** 35 = 29 + 3 + 3 = 21 + 11 + 3 |
| **15.** 34 = 31 + 3 = 29 + 5 | **15.** 37 = 31 + 3 + 3 = 29 + 5 + 3 |
| **16.** 36 = 31 + 5 = 29 + 7 | **16.** 39 = 31 + 5 + 3 = 29 + 7 + 3 |
| **17.** 38 = 31 + 7 | **17.** 41 = 31 + 7 + 3 |
| **18.** 40 = 37 + 3 = 29 + 11 | **18.** 43 = 37 + 3 + 3 = 29 + 11 + 3 |
| **19.** 42 = 37 + 5 = 29 + 13 | **19.** 45 = 37 + 5 + 3 = 29 + 13 + 3 |
| **20.** 44 = 37 + 7 = 31 + 13 | **20.** 47 = 37 + 7 + 3 = 31 + 13 + 3 |
| **21.** 46 = 41 + 5 = 29 + 17 | **21.** 49 = 41 + 5 + 3 = 29 + 17 + 3 |
| **22.** 48 = 43 + 5 = 29 + 19 | **22.** 51 = 43 + 5 + 3 = 29 + 19 + 3 |
| **23.** 50 = 43 + 7 = 31 + 19 | **23.** 53 = 43 + 7 + 3 = 31 + 19 + 3 |
| **24.** 52 = 47 + 5 = 29 + 23 | **24.** 55 = 47 + 5 + 3 = 29 + 23 + 3 |
| **25.** 54 = 47 + 7 = 31 + 23 | **25.** 57 = 47 + 7 + 3 = 31 + 23 + 3 |

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## Strong Conjecture Partitions Constructed from Weak Conjecture Partitions

(Consecutive Descendants)

The following table contains partition equations for both the weak conjecture and the strong conjecture. The strong conjecture partitions on the right were obtained from **original** weak conjecture partitions on the left by subtracting a 3 from both sides of each partition equation on the left. Note that the partitions on the right are not all exactly as those on the left of the previous Table 3, since Table 4 is the reversal of Table 3, only in principle. Both sides of the table show that the partition production process will not terminate and can continue indefinitely. Thus, for the weak conjecture, every odd integer greater than 7 can be expressed as the sum of three odd primes; and for the strong conjecture, every even integer greater than 4 can be expressed as the sum of two odd primes. For the weak conjecture, one will begin with the partition equation $9 = 3 + 3 + 3$, and apply the addition of 2 to both sides of the equation to produce the partition for the next odd integer, 11. From the partition equation, $11 = 5 + 3 + 3$, one will repeat the 2-addition process to obtain the partition for the next odd integer, 13 From the partition for 13, the process can continue indefinitely. For the strong conjecture, one will subtract a 3 from both sides of each corresponding partition equation of the weak conjecture column. One will note that it is much easier to convert a strong conjecture partition to a weak conjecture partition than to convert a weak conjecture partition to a strong conjecture partition. Below, only the bold-face to the right are the strong partitions.

**Table 4**

| **Weak Goldbach Conjecture** | **Strong Goldbach Conjecture** |
|---|---|
| **1.** $9 = 3 + 3 + 3$ ⇓ Begin | **1.** $6 = \mathbf{3} + \mathbf{3}+$ ⇓ Begin |
| **2.** $11 = 5 + 3 + 3$ | **2.** $8 = \mathbf{5} + \mathbf{3}$ |
| **3.** $13 = 7 + 3 + 3$ | **3.** $10 = \mathbf{7} + \mathbf{3}$ |
| **4.** $15 = 7 + 5 + 3$ | **4.** $12 = \mathbf{7} + \mathbf{5}$ |
| **5.** $17 = 7 + 7 + 3$ | **5.** $14 = \mathbf{7} + \mathbf{7}$ |
| **6.** $19 = 7 + 7 + 5$ | **6.** $16 = 7 + 4 + 5 = \mathbf{11} + \mathbf{5}$ |
| **7.** $21 = 7 + 7 + 7$ | **7.** $18 = 7 + 7 + 4 = \mathbf{7} + \mathbf{11}$ |
| **8.** $23 = 11 + 5 + 7$ | **8.** $20 = 11 + 5 + 4 = \mathbf{13} + \mathbf{7}$ |
| **9.** $25 = 13 + 5 + 7$ | **9.** $22 = 13 + 5 + 4 = \mathbf{17} + \mathbf{5}$ |
| **10.** $27 = 13 + 7 + 7$ | **10.** $24 = 13 + 7 + 4 = \mathbf{13} + \mathbf{11}$ |
| **11.** $29 = 17 + 5 + 7$ | **11.** $26 = 17 + 5 + 4 = \mathbf{19} + \mathbf{7}$ |
| **12.** $31 = 19 + 5 + 7$ | **12.** $28 = 19 + 5 + 4 = \mathbf{23} + \mathbf{5}$ |
| **13.** $33 = 19 + 7 + 7$ . | **13.** $30 = 19 + 7 + 4 = \mathbf{19} + \mathbf{11}$ . |
| **14.** $35 = 23 + 5 + 7$ | **14.** $32 = 23 + 5 + 4 = \mathbf{29} + \mathbf{3}$ |
| **15.** $37 = 23 + 7 + 7$ | **15.** $34 = 23 + 7 + 4 = \mathbf{29} + \mathbf{5}$ |
| **16.** $39 = 23 + 11 + 5$ . | **16.** $36 = 23 + 11 + 2 = \mathbf{23} + \mathbf{13}$ . |
| **17.** $41 = 23 + 13 + 5$ | **17.** $38 = 23 + 13 + 2 = \mathbf{31} + \mathbf{7}$ |
| **18.** $43 = 23 + 13 + 7$ | **18.** $40 = 23 + 13 + 4 = \mathbf{23} + \mathbf{17}$ |
| **19.** $45 = 29 + 13 + 3$ | **19.** $42 = \mathbf{29} + \mathbf{13}$ |
| **20.** $47 = 29 + 13 + 5$ | **20.** $44 = 29 + 13 + 2 = \mathbf{31} + \mathbf{13}$ |
| **21.** $49 = 29 + 13 + 7$ | **21.** $46 = 29 + 13 + 4 = \mathbf{29} + \mathbf{17}$ |
| **22.** $51 = 31 + 13 + 7$ | **22.** $48 = 31 + 13 + 4 = \mathbf{31} + \mathbf{17}$ |
| **23.** $53 = 31 + 17 + 5$ | **23.** $50 = 31 + 17 + 2 = \mathbf{31} + \mathbf{19}$ |
| **24.** $55 = 31 + 17 + 7$ | **24.** $52 = 31 + 17 + 4 = \mathbf{41} + \mathbf{11}$ |
| **25.** $57 = 31 + 19 + 7$ | **25.** $54 = 31 + 19 + 4 = \mathbf{31} + \mathbf{23}$ |
| **26.** $59 = 31 + 17 + 11$ | **26.** $56 = 31 + 17 + 8 = \mathbf{37} + \mathbf{19}$ |

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# Non-Consecutive Partition Production

The following examples will show that the partition production does not have to be consecutive.

**Finding partitions for 84, 100, 1000,** 372,131,740 **, and** 400,000,001,000 **and others**

## Strong Goldbach Conjecture

**1. From 6 to 84**

6 = 3 + 3
6 + (84 – 6) = 3 + (3 + 84 – 6)
6 + (78) = 3 + (3 + 78)
84 = 3 + 81 (Note : 81 is not prime)
84 = (3 + 2) + (81 – 2)
**84 = 5 + 79** or **79 + 5**. Note: 79 is prime

(---Note above that if one adds 2 to 81, one obtains 83 which is prime, but on subtracting 2 from 3, one would get 1, which is not prime)

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**2. From 6 to 100**

6 = 3 + 3
6 + (100 – 6) = 3 + (3 + 100 – 6)
6 + (94) = 3 + (3 + 94)
**100 = 3 + 97** **Note** : 97 is prime.

**3a. From 6 to 1000**

6 = 3 + 3
6 + (1000 – 6) = 3 + (3 + (1000 – 6)
6 + 994 = 3 + (3 + 994)
1000 = 3 + (3 + 994)
**1000 = 3 + 997** Note : 997 is prime

**3b From 84 to 6**

84 = 5 + 79
84 + (6 – 84) = 5 + (79 + (6 – 84))
84 – 78 = 5 + 1 Note : 1 is not prime
**6 = 3 + 3** (Add 2 to 1 and subtract 2 from 5)

**3c From 92 to 6**

92 = 3 + 89
92 + (6 – 92) = 3 + (89 + 6 – 92)
92 – 86 = 3 + 89 – 86
**6 = 3 + 3**

## Weak Goldbach Conjecture

**From 9 to 101**

**A** 9 = 3 + 3 + 3
9 + (101 – 9) = 3 + 3 + [(3 + (101 – 9)]
9 + 92 = 3 + 3 + (3 + 92)
101 = (3 + 4) + (3 + 2) + (95 – 6)
**101 = 7 + 5 + 89.** Note: 89 is prime

**From 29 to 53**

**B** 29 = 17 + 5 + 7
29 + (53 – 29) = 17 + 5 + (7 + (53 – 29)))
29 + 24 = 17 + 5 + (7 + 24)
**53 = 17 + 5 + 31**

**From 53 to 29**

**C** 53 = 17 + 5 + 31
53 + (29 – 53) = 17 + 5 + (31 + (29 – 53))
53 – 24 = 17 + 5 + (31 – 24)
**29 = 17 + 5 + 7**

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**From 101 to 9**

**D** 101 = 7 + 5 + 89
101 + (9 – 101) = 7 + 5 + (89 + (9 – 101)
101 – 92 = 7 + 5 + (89 – 92)
9 = 7 + 5 + (89 – 92)
9 = 7 + 5 – 3
9 = (7 – 4) + (5 – 2) + (–3 + 6)
**9 = 3 + 3 + 3**

## Strong Goldbach Conjecture

**4. From 6 to 372,131,740**

**4.**
Three hundred seventy-two million,one hundred thirty-one thousand, seven hundred,forty
$6 + (372,131,740 - 6) = 3 + (3 + (372,131,740 - 6)$
$6 + (372,131,734) = 3 + (3 + (372,131,734)$
**$372,131,740 = 3 + 372,131,737$**
**Note** : 372,131,737 is prime

**5. From 6 to 400,000,001,000**

**5.** Four hundred billion, one thousand
$6 = 3 + 3$
$6 + (400,000,001,000 - 6) =$
$3 + (3 + 400,000,001,000 - 6)$
$6 + (400,000,000,994) =$
$3 + (3 + (400,000,000,994)$
**$400,000,001,000 = 3 + 400,000,000,997$**
**Note** : 400,000,000,997 is prime

## Weak Goldbach Conjecture

**From 9 to 1001**

**E** $9 = 3 + 3 + 3$
$9 + (1001 - 9) = 3 + 3 + [(3 + (1001 - 9)]$
$9 + 992 = 3 + 3 + (3 + 992)$
$9 + 992 = 3 + 3 + 995$
$1001 = (3 + 2) + (3 + 2) + 991$
$1001 = 5 + 5 + 991$
Note: 991 is prime

**From 9 to** 400,000,001,001

**F** $9 = 3 + 3 + 3$
$9 + (400,000,001,001 - 9) =$
$3 + 3 + [(3 + (400,000,001,001 - 9)]$
$9 + 400,000,000,992 =$
$3 + 3 + (3 + 400,000,000,992)$
$9 + 400,000,000,992 =$
$3 + 3 + 400,000,000,995$
$400,000,001,001 =$
$3 + 3 + (400,000,000,963 + 32)$
$400,000,001,001 =$
$19 + 19 + 400,000,000,963$.

On the next page, one will generalize the non-consecutive partition production process to show that every even integer, $E_n$ $(n > 4)$can be expressed as the sum of two odd primes; and also that every odd integer, $O_n$ $(n > 7)$ can be expressed as the sum of three odd primes.

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# Strong & Weak Goldbach Conjectures Proved Side-by-Side

## Strong Goldbach Conjecture

**Given**: **1**. A known Goldbach partition of the even integer $E_1 = P_1 + P_2$, where $P_1$ and $P_2$ are prime odd integers.

**2. The** even integer, $E_n, (n > 4)$ whose partition is to be determined..

**Required:** To show that the even integer $E_n$ has a Goldbach partition

**Plan:** Let $E_n = P_r + P_s$, where $P_r$ and $P_s$ are prime odd integers. The proof would be complete after finding a general formula for $E_n$ for a Goldbach partition equation.

**Statements**

**1**. $E_1$ is an even integer (given)

**2**. $P_1$ and $P_2$ are prime odd integers

**3.** $E_1 = P_1 + P_2$, (Given)

**4.** $E_1 + (E_n - E_1) = P_1 + (P_2 + (E_n - E_1))$

**5.** $E_n = P_1 + (P_2 + E_n - E_1)$

**6.** In statement 5, $P_1$ is an odd prime integer

**7. Case A**: $(P_2 + E_n - E_1)$ is prime (desirable),

$P_r = P_1$ and $P_s = (P_2 + E_n - E_1)$

$\boxed{E_n = P_r + P_s = P_1 + (P_2 + E_n - E_1)}$;

and the proof is complete.

**Case B:** $(P_2 + E_n - E_1)$ is **not** prime.

The addition or subtraction of a 2 or 2 's would make $(P_2 + E_n - E_1)$ become prime, However, the 2 or 2's added or subtracted must be subtracted from or added to $P_1$.

**Case B1**: $P_1 = 3$; $p_s = (P_2 + E_n - E_1) - 2t$;

$p_r = 3 + 2t$, where $t$ is the number of times 2 is subtracted before primality occurs.

$E_n = P_r + P_s = \boxed{(3 + 2t) + [(P_2 + E_n - E_1) - 2t]}$

**Case B2:** $P_1 > 3$; $P_s = (P_2 + E_n - E_1) \pm 2t$ and $P_r = P_1 \mp 2t$. After the inevitable successful changes in $P_1$ and $(P_2 + E_n - E_1)$,

$E_n = P_r + P_s =$

$\boxed{(P_1 \pm 2t) + [(P_2 + E_n - E_1) \mp 2t]}$,

## Weak Goldbach Conjecture

**Given**: **1**. A known Goldbach partition of the odd integer $O_1 = P_1 + P_2 + P_3$, where $P_1, P_2$, and $P_3$ are prime odd integers. **2**. The odd integer, $O_n$, $(n > 7)$ whose partition is to be determined..

**Required:** To show that the odd integer $O_n$ has a Goldbach partition ,

**Plan:** Let $O_n = P_r + P_s + P_t$, where $P_r$, $P_s$ and $P_t$ are prime odd integers. The proof would be complete after finding a general formula for $O_n$ for a Goldbach partition equation.

**Statements**

**1.** $O_n$ is an odd integer (given)

**2**. $P_1$, $P_2$ and $P_3$ are prime odd integers

**3.** $O_1 = P_1 + P_2 + P_3$

**4.** $O_1 + (O_n - O_1) = P_1 + P_2 + [P_3 + (O_n - O_1)]$

**5.** $O_n = P_1 + P_2 + [P_3 + (O_n - O_1)]$

**6.** In statement 5, above, $P_1$, $P_2$ are prime

**7. Case A**: $[P_3 + (O_n - O_1)]$ is prime (desirable),

$P_r = P_1$, $P_s = P_2$, $P_t = [P_3 + (O_n - O_1)]$

$O_n = P_r + P_s + P_t = \boxed{P_1 + P_2 + [P_3 + (O_n - O_1)]}$;

and the proof is complete.

**Case B:** $[P_3 + (O_n - O_1)]$ is **not** prime.

The addition or subtraction of a 2 or 2 's would make $[P_3 + (O_n - O_1)]$ become prime, However, the 2 or 2's added or subtracted must be subtracted from or added to $P_1$. or $P_2$

**Case B1** $P_1 = 3$; $P_2 = 3$, $p_t = [P_3 + (O_n - O_1)] - 2t$;

$p_r = 3 + 2t_1$, $p_s = 3 + 2t_2$, where $2t_1 + 2t_2 = 2t$ and $2t_1$ or $2t_2$ may be zero but not both and where $t$ is the number of times 2 is subtracted before primality occurs

$\boxed{O_n = (3 + 2t_1) + (3 + 2t_2) + [P_3 + (O_n - O_1)] - 2t}$

**Case B2:** $P_1 > 3$, $P_2 > 3$; $P_r = (P_1 \mp 2t_1)$,

$P_s = (P_2 \mp 2t_2)$; $P_t = [P_3 + (O_n - O_1) \pm 2t]$ & where $2t_1 + 2t_2 = 2t$ and $2t_1$ or $2t_2$ may be zero but not both. .

After the changes in $P_1$, $P_2$ and $[P_3 + (O_n - O_1)]$

$\boxed{O_n = (P_1 \mp 2t_1) + (P_2 \mp 2t_2) + [P_3 + (O_n - O_1) \pm 2t]}$



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**Strong Goldbach Conjecture**

As $n \to \infty$, the general Goldbach partition equations for $E_n$. would still be given by the above equations, which will always be defined for all positive integers and never zero, and where $E_1, P_1, P_2$, are from any known Goldbach partition equation.

Therefore, every even integer, $E_n$ $(n > 4)$can be expressed as the sum of two odd primes; and the proof is complete.

**Weak Goldbach Conjecture**

As $n \to \infty$, the general Goldbach partition equations, above, for $O_n$, would still be given by the above equations which would always be defined for all positive integers, and never zero; and where $O_1, P_1, P_2$, and $P_3$ are from a known Goldbach partition equation. Therefore, every odd integer, $O_n$ $(n > 7)$ can be expressed as the sum of three odd primes; and the proof is complete.

# Discussion

**Strong Goldbach Conjecture**

Generally, $\boxed{E_n = P_1 + (P_2 + E_n - E_1)}$, where $E_1, P_1, P_2$, are from any known Goldbach partition $E_1 = P_1 + P_2$.

If $(P_2 + E_n - E_1)$ is prime, there is no more work to be done, and one would have found a Goldbach partition for the even number, $E_n$; However, if it is not prime, one has to apply cases B1 and B2, above. For example, in the table for Condensed Goldbach Partition Production (p.4), one can obtain Goldbach partition # 36 from partition #48 and vice versa. Thus by applying the derived formula, one can obtain a Goldbach partition for any even integer greater than 4. For examples on Case A of the proof, see Example 2, 3a, 4 and 5 on page 8-9. For Case B1, see Example 1, p.8. Interestingly, one can also obtain the partition equation $6 = 3 + 3$ from any other partition equation, as in examples 3b and 3c, p.8 . Such a result is very convincing that every even integer can be written as the sum of two odd primes. Thus, given a Goldbach partition of an even integer, by hand, one can find, without exception, a Goldbach partition for any other even integer, within minutes.

**Weak Goldbach Conjecture**

Generally, $\boxed{O_n = P_1 + P_2 + [P_3 + (O_n - O_1)]}$, where $O_1$, $P_2, P_3$ are from any known Goldbach partition $O_1 = P_1 + P_2 + P_3$. If $[P_3 + (O_n - O_1)]$ is prime, there no more work to be done, and one would have found a Goldbach partition for the odd integer $O_n$. However, if it is not prime, one has to apply cases B1 and B2, above. It is to be noted for example that in the table on page 5, one can obtain Goldbach partition #23 from partition #11 and vice versa as illustrated on page 7 Box B-C

For examples on Case A of the proof, see Example B, p. 8. For Case B1, see Example B, p.8. For Case B2, see Example A, p.8 Interestingly, one can also obtain the partition equation $9 = 3 + 3 + 3$ from any other partition equation, as in example Box D, p.8. Such a result is very convincing that every odd integer can be written as the sum of three odd primes. Thus, given a Goldbach partition of an odd integer, by hand, one can find, without exception, a Goldbach partition for any other odd integer, within minutes.

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# Comparison of Compound Interest Calculations & Goldbach Partition Productions

As shown below, the repetitive process in the consecutive partition production is similar to the repetitive process in compound interest calculations, except in the operations involved..

## Compound Interest Calculations

**Example:**
Find the interest and the amount at the end of each year on a $60 loan at interest rate of 5% compounded annually for 3 years.

**Method 1:**

Interest for the first year = $\frac{5}{100} \times \frac{\$60}{1} = \$3$

Amount at the end of the 1st year = $60 + 3 = $63 .

Interest for the 2nd year = $\frac{5}{100} \times \frac{\$63}{1} = \$3.15$

Amount at the end of the 2nd year = $63 + $3.15 = $66.15

Interest for the 3rd year = $\frac{5}{100} \times \frac{\$66.15}{1} = \$3.31$

Amount at the end of the 3rd year = $66.15 + $3.31 = $69.46

Note above that the amount at the end of each year is the principal for the following year.

Also, Amount = principal + interest

Amount at the end of the first year = $60 + 3

Amount at the end of the second year = $63 + $3.15

Amount at the end of the third year = $66.15 + $3.31

Let A = the amount, P = the principal and I = the interest

Then, $A = P + I$

Let $A_1$ = the amount at the end of the **first** year.

Let $A_2$ = the amount at the end of the **second** year.

Let $A_3$ = the amount at the end of the **third** year.

Then, from above,

$A_1$ = $63 = $60 + $3

$A_2$ = $66.15 = $63 + $3.15

$\boxed{A_3 = \mathbf{\$69.46} = \$66.15 + \$3.31}$

**Note** above and Method 2 that the amount calculated using the compound interest formula is the same as $A_3$ (by the repetitive process). .One may say, using real numbers, that the partition equations for $A_1$, $A_2$ and $A_3$ are

$A_1$ = $63 = $60 + $3;,

$A_2$ = $66.15 = $63 + $3.15 and

$A_3$ = **$69.46** = $66.15 + $3.31

**Method 2:** By the compound interest formula, $A = P(1 + r)^t$ where $A$ = the amount, $P$ is the principal, $r$ = interest rate per year, $t$ = number of years.

Substituting accordingly in

$A = P(1 + r)^t$

$A = \$60(1.05)^3$

$= \$(60)(1.16)$

$\boxed{= \mathbf{\$69.46}}$

## Goldbach Partition Productions

**Example:**
Given the Goldbach partition equation $6 = 3 + 3$, find a Goldbach partition equation for the even integer, 10.

**Method 1** (by the 2-addition)

**1**. $6 = 3 + 3$ ⇓ Begin

**2.** $8 = 5 + 3$ (add 2 to 6, add 2 to 1st 3)

$\boxed{\textbf{3}.\ 10 = 7 + 3}$ (add 2 to 8,add 2 to the 5)

**Method 2**

Applying a **formula** derived on page 10, Case A of the proof:

$E_n = P_1 + (P_2 + E_n - E_1)$

$P_1 = 3,, P_2 = 3,\ E_1 = 6, E_n = 10,$

$E_{10} = P_1 + (P_2 + E_n - E_1)$

$10 = 3 + (3 + 10 - 6)$

$10 = 3 + (3 + 4)$

$\boxed{10 = 3 + 7}$

One can observe from above that Method 1 for both the consecutive partition production and the compound interest calculation is a repetitive process, while Method 2 is by a formula. The author generalizes that any math problem that exhibits the above repetitive behavior can be handled similarly...

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# Conclusion

## Strong Goldbach Conjecture

It has been shown in this paper that every even integer greater than 4 can be expressed as the sum of two odd primes. The proof was guided by the approach the author used in finding Goldbach partitions. The approach for finding the partitions led directly to evidence that every even integer greater than 4 is the sum of two odd primes. The main principle for finding Goldbach partitions from a known partition is the application of the addition axiom to a Goldbach partition equation. It was shown that given an equation for a Goldbach partition, one can produce a Goldbach partition for any even integer greater than 4.

Beginning with the partition equation, $6 = 3 + 3$, and applying the addition of a 2 to both sides of this equation sequentially and repeatedly, one obtained Goldbach partitions for over 180 consecutive even integers. Also, a derived formula was used successfully to find partitions for the non-consecutive even integers, 100; 1000; 372,131,740; and 400,000,001,1000.

In addition to directly producing partitions of the strong conjecture, one can also produce partitions of the strong conjecture from the partitions of the weak conjecture. It is to be noted from the previous page that the repetitive process in partition production is similar to the repetitive process in compound interest calculations, except for the operations involved. Such similarity contributes to the validity of the approach used in showing that the conjectures are true.

It is concluded that, knowing a single Goldbach partition equation for an even integer, one can by hand, find a Goldbach partition quickly for any other even integer.

## Weak Goldbach Conjecture

It has been shown in this paper that every odd integer greater than 7 can be expressed as the sum of three odd primes. The proof was guided by the approach the author used in finding Goldbach partitions. The approach for finding the partitions led directly to evidence that every odd integer greater than 7 is the sum of three odd primes. The main principle for finding Goldbach partitions from a known partition is the application of the addition axiom to a Goldbach partition equation. It was shown that given an equation for a Goldbach partition, one can produce a Goldbach partition for any odd integer greater than 7. Three tables for the production of Goldbach partition equations were covered. The first table began with the partition equation, $9 = 3 + 3 + 3$, and applying the addition of a 2 to both sides of this equation sequentially and repeatedly, one obtained Goldbach partitions for 26 consecutive odd integers. Also, a derived formula was used successfully to find partitions for the non-consecutive odd integers, 1001; 10001; and 400,000,001,1001. The second table was constructed from a two-prime integer partitions by mere addition of a 3 to both sides of the partition equations.

In addition to directly producing partitions of the weak conjecture, one can also produce partitions of the weak conjecture from the partitions of the strong conjecture.

It is concluded that, knowing a single Goldbach partition equation for an odd integer, one can by hand, find a Goldbach partition quickly for any other odd integer.

## References

1. See also: Goldbach Conjecture Proved Remarkably at viXra:2006.0226.

**2.** The approach used in covering Goldbach conjecture in this paper is similar to the approach the author used in proving Beal Conjecture (vixra: 2001.0694).

**3.** In covering the Goldbach conjecture, one must have quick access to the list of the prime numbers, Some places on the web for the lists of prime numbers are at: 1.www.mathsfun.com; 2. CalculatorSoup.com.

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# Extra
## Integer Humor

## 1. Conversation Between the Even Integers 6 and 84

**Mr. 84 speaks:** Mr. 6. I can see that you have a Goldbach partition. Can you help me get my own Goldbach partition?

**Mr. 6 answers:** Yes, I can help you using my own Goldbach partition as in box **A** below:

| **A** | **B** |
|---|---|
| $6 = 3 + 3$<br>$6 + (84 - 6) = 3 + (3 + 84 - 6)$<br>$6 + (78) = 3 + (3 + 78)$<br>$84 = 3 + 81$ (Note : 81 is not prime)<br>$84 = (3 + 2) + (81 - 2)$<br>**$84 = 5 + 79$** or **$79 + 5$**. Note: 79 is prime | $84 = 5 + 79$<br>$84 + (6 - 84) = 5 + (79 + (6 - 84))$<br>$84 - 78 = 5 + 1$ Note : 1 is not prime<br>**$6 = 3 + 3$** (Add 2 to 1 and subtract 2 from 5) |

**Mr. 84 speaks**: Thank you Mr. 6.

A week later, Mr. 84 meets Mr. 6 and Mr. 6 has no Goldbach partition.

**Mr. 84 speaks:** Mr. 6, where is your Goldbach partition?

**Mr. 6 answers** I lost my partition.

**Mr. 84 speaks** without hesitation: Mr. 6, I can help you get your Goldbach partition back as in box **B** above.

**Mr. 6 speak**s: Thank you very much, Mr. 84. I got my Goldbach partition back.

**Mr. 84 speaks:** Don't mention. You were kind to me, the first time we met.

**Mr. 84 speaks:** Here, comes my neighbor, Mr. 86, without a Goldbach partition. From my partition, I can easily get Mr. 86 a partition as in box **C** below

| **C** |
|---|
| $84 = 5 + 79$<br>$84 + 2 = (5 + 2) + 79$<br>**$86 = 7 + 79$** (Add 2 to 84 and add 2 to 5) |

**Mr. 86 speak**s: Thank you Mr. 84. I have a Goldbach partition for the first time.

**Mr. 6 speak**s to Mr. 84:: I can see from your partitions that you and Mr. 86 live on the same street, 79th street.

**Mr. 84 speaks:** Yes. Mr. 86 is a good neighbor.

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## 2. Conversation Between the Odd Integers 29 and 53

**Mr. 53 speaks:** Mr. 29. I can see that you have a Goldbach partition. Can you help me get my own Goldbach partition?

**Mr. 29 answers:** Yes, I can help you using my own Goldbach partition as in box **A** below:

| **A** $29 = 17 + 5 + 7$ | **B** $53 = 17 + 5 + 31$ |
|---|---|
| $29 + (53 - 29) = 17 + 5 + (7 + (53 - 29)))$ | $53 + (29 - 53) = 17 + 5 + (31 + (29 - 53))$ |
| $29 + 24 = 17 + 5 + (7 + 24)$ | $53 - 24 = 17 + 5 + (31 - 24)$ |
| $\mathbf{53 = 17 + 5 + 31 =}\ 5 + 17 + 31$ | $\mathbf{29 = 17 + 5 + 7 = 5 + 7 + 17}$ |

**Mr. 53 speaks**: Thank you Mr. 29.

A week later, Mr. 53 meets Mr. 29 and Mr. 29 has no Goldbach partition.

**Mr. 53 speaks:** Mr. 29, where is your Goldbach partition?

**Mr. 29 answers:** I lost my partition.

**Mr. 53 speaks** without hesitation: Mr. 29, I can help you get your Goldbach partition back as in box **B** above.

**Mr. 29 speak**s: Thank you very much, Mr. 53. I got my Goldbach partition back.

**Mr. 53 speaks:** Don't mention. You were kind to me, the first time we met.

**Mr. 53 speaks:** Here, comes my neighbor, Mr. 51, without a Goldbach partition. From my partition, I can easily get Mr. 51 a partition as in box **C** below.

| **C** $53 = 5 + 17 + 31$ |
|---|
| $53 + (51 - 53) = 5 + 17 + (31 + 51 - 53)$ |
| $53 - 2 = 5 + 17 + (31 - 2))$ |
| $\mathbf{51 = 5 + 17 + 29}$ |

**Mr. 51 speak**s: Thank you Mr. 53. I have a Goldbach partition for the first time.

**Mr. 29 speak**s to Mr. 51: I can see from our partitions that my name is the same as your year of birth, 29.

**Mr. 51 speaks:** Yes. I was born in 1929.



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## 3. Conversation Between the Even Integer 6 and the Odd Integer 9

**Mr. 9 speaks:** Mr. 6. I can see that you have a Goldbach partition. Can you help me get my own Goldbach partition?

**Mr. 6 answers:** Yes, I can easily help you using my own Goldbach partition as in box **A** below:

| | |
|---|---|
| **A** $6 = 3 + 3$<br>$6 + 3 = 3 + 3 + 3$<br>$\mathbf{9 = 3 + 3 + 3}$ | **B** $9 = 3 + 3 + 3$<br>$9 - 3 = 3 + 3 + 3 - 3$<br>$\mathbf{6 = 3 + 3}$ |

**Mr. 9 speaks**: Thank you Mr. 6.

A week later, Mr. 9 meets Mr. 6 and Mr. 6 has no Goldbach partition.

**Mr. 9 speaks:** Mr. 6, where is your Goldbach partition?

**Mr. 6 answers** I lost my partition.

**Mr. 9 speaks** without hesitation: Mr. 6, I can help you get your Goldbach partition back as in box **B** above.

**Mr. 6 speak**s: Thank you very much, Mr.. 9. I got my Goldbach partition back.

**Mr. 9 speaks:** Don't mention. You have been kind to people without, Goldbach partitions.

**Mr. 9 speaks:** Here, comes my neighbor, Mr.11, without a Goldbach partition. From my partition, I can easily get Mr. 11 a partition as in box **C** below

| | |
|---|---|
| **C** $9 = 3 + 3 + 3$<br>$9 + 2 = 5 + 3 + 3$<br>$\mathbf{11 = 5 + 3 + 3}$ | (Add 2 to 9 and add 2 to the first 3) |

**Mr. 11 speak**s: Thank you Mr. 9. I have a Goldbach partition for the first time.

**Mr. 6 speak**s:: I can see from our partitions that each one of us has at least two 3's in his partition

**Mr. 9 speaks:** Yes. 3 + 3.

**Mr. 11 speaks:** From my observation so far, it seems that every even integer greater than 4 or every odd integer greater than 7 can obtain a Goldbach partition from anyone with a Goldbach partition.

**Mr. 6 and Mr. 9** respond in unison :Yes, Mr. **11.** You are right.

**Adonten**



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